Math 323 - Formal Mathematical Reasoning and Writing

Problem Session

Wednesday, 4/22/15

1. Construct a function that is:
   (a) Injective, but not surjective.
   (b) Surjective, but not injective.
   (c) Neither injective nor surjective.
2. [1] Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = 2n$, and let $g : \mathbb{Z} \to \mathbb{Z}$ be defined by
$$g(m) := \begin{cases} \frac{m}{2} & \text{if } m \text{ is even} \\ 34 & \text{if } m \text{ is odd} \end{cases}.$$
   (a) Calculate $(g \circ f)(n)$.
   (b) Is $g(n) = f^{-1}(n)$?
3. The following statements are false.
   (i) If $f : A \to B$ and $g : B \to C$ are functions and $g \circ f$ is injective, then f is injective and g is injective.
   (ii) If $f : A \to B$ and $g : B \to C$ are functions and $g \circ f$ is surjective, then f is surjective and g is surjective.
   (a) How would you *show* that these statements are false?
   (b) Correct the statements so they are true.
4. [2] Let $A = \{1, 2, 3\}$. Let $\mathrm{Bij}(A) = \{f : A \to A \mid f(x) \text{ is a bijection}\}$. Define a relation on $\mathrm{Bij}\, A$ by $f \sim g$ if and only if
$$\exists b \in \mathrm{Bij}(A) \text{ such that } g = b^{-1} \circ f \circ b$$
   (a) List all the elements of $\mathrm{Bij}(A)$ by drawing them as arrow diagrams.
   (b) Prove that the relation $\sim$ defined above is an equivalence relation.
   (c) Find $\mathrm{Bij}(A)/\sim$.

[1] `http://math.sfsu.edu/federico/Clase/Math301.S10/book.pdf`, Example 9.8
[2] Madden §11.5 #7